Introduction to computational plasticity using FORTRAN

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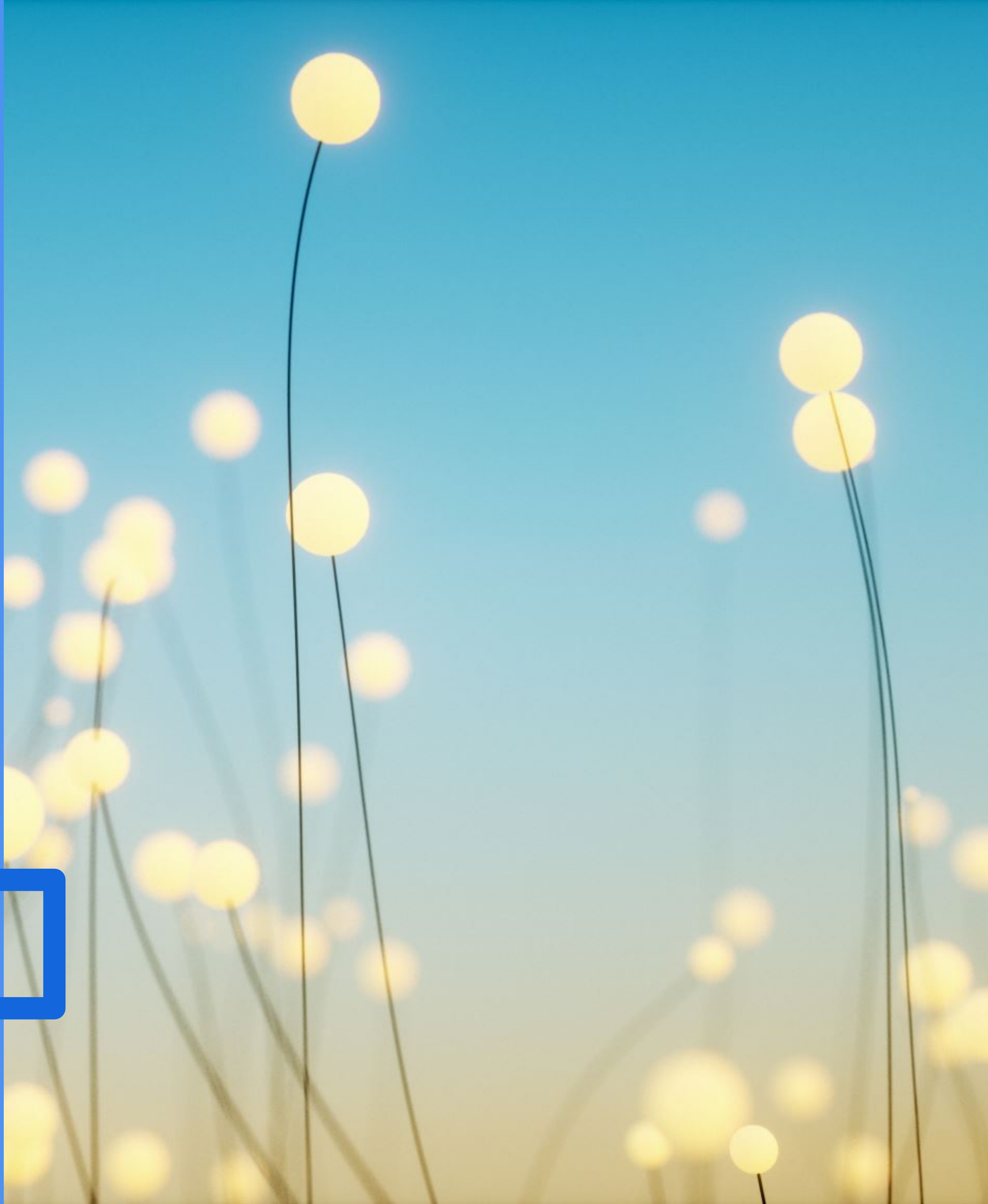
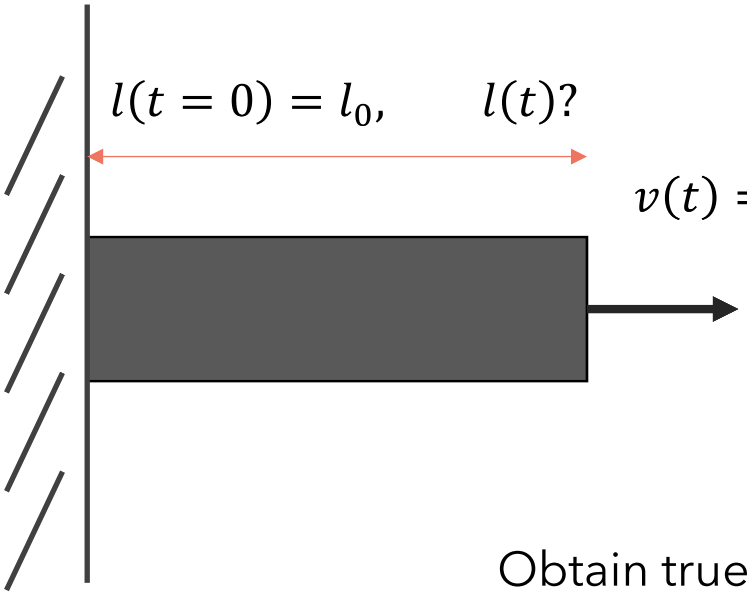


Stretching a one-dimensional rod by a fixed speed for a certain time



$$v(t) = 0\times t + c = c \qquad \frac{dv}{dt} = 0$$

Obtain true strain as function of time $\varepsilon(t)$

Extension – fundamental solution

- Suppose we have initial length l_0 [mm], which undergoes extension at a constant rate c.
- The above can be mathematically expressed as $$l \equiv l(t) \text{ with } l(t=0) = l_0 \text{ and } \frac{dl}{dt} = c$$
- Express l as function of t: $l(t)$?

$$l(t) = l_0 + \int_0^t dl \quad = l_0 + \int_0^t \frac{dl}{dt} dt$$

$$= l_0 + \int_0^t c dt = l_0 + ct \text{ [mm]}$$

True strain as function of time

$$d\varepsilon = \frac{dl}{l}$$

Since $l \equiv l(t)$,

$$d\varepsilon(t) = \frac{dl}{l(t)}$$

As such,

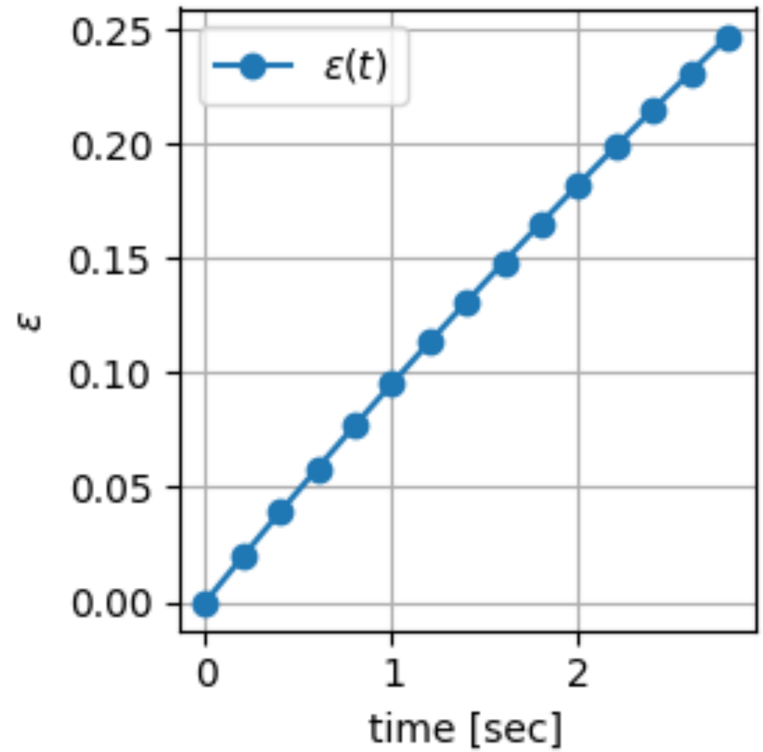
$$\varepsilon(t) = \varepsilon(t=0) + \int_0^t d\varepsilon = 0 + \int_0^t \frac{dl}{l}$$

$$= \int_0^t \frac{1}{l}\frac{dl}{dt}dt = c\int_0^t \frac{1}{l_0 + ct}dt = c\left[\frac{1}{c}\ln(l_0 + ct)\right]_0^t = \ln(l_0 + ct) - \ln(l_0)$$

Program to calculate $\varepsilon(t)$

Although the below works as it is, it contains a bug – try the program by changing the initial value l0 to 5. Find the bug and address the issue.

```
c     U as a function of time
      program et
      implicit none
      real l0, vel, dt, t, eps
c     inputs
      l0=1                      ! [mm]
      vel=0.1                   ! [mm/s]
      dt=0.2
      t=0.
c     --
      open(1,file='et.txt')
      do while(t.lt.3)
         write(1,'(2f10.5)')t,eps(l0,vel,t)
         t=t+dt
      enddo
      close(1)
      end program
c     ---------------------------
c     analytical function eps as function of t
      real function eps(l0,vel,t)
      implicit none
      real vel,t,l0
      eps=log(l0+vel*t)
      return
      end function
```



Result of the left

Extension – Numerical solution (Euler)

- Suppose we have initial length = 1 [mm], which undergoes extension of 0.1 [mm/s].
- Numerical strategy to solve the above:

$$\mathrm{d}\varepsilon = \frac{\mathrm{d}l}{l}$$

$$\Delta\varepsilon = \frac{\Delta l}{l}$$

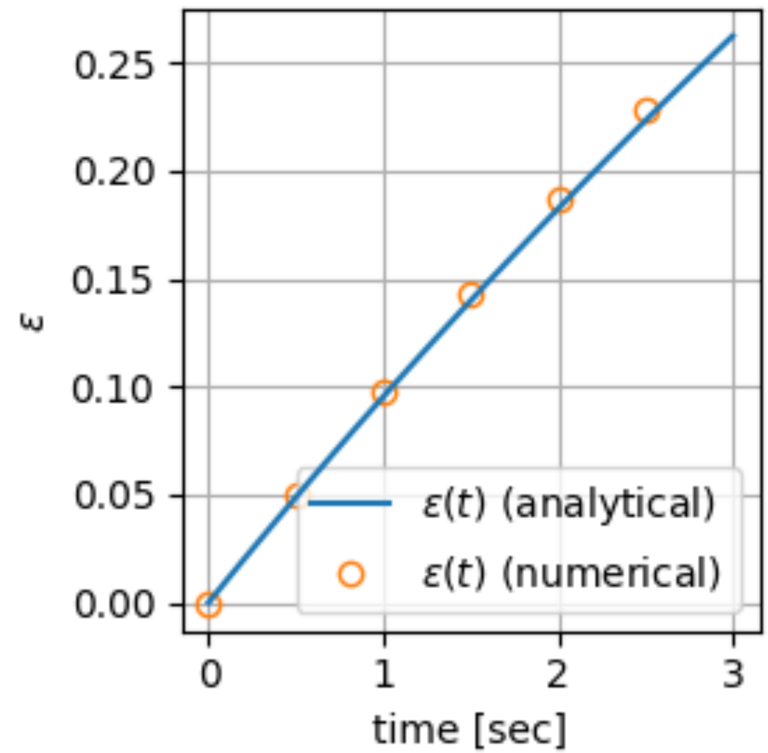
$$\varepsilon_{(\mathrm{n+1})} = \varepsilon_{(\mathrm{n})} + \Delta\varepsilon$$

$$\Delta \mathrm{l} = \frac{\mathrm{dl}}{\mathrm{dt}}\Delta t$$

With sufficiently small Δt, the convergence should be attained.

Program to numerically estimate e(t)

```
c      Numerical solution to find E as function of time

       program et_num
       implicit none
       real dt, t, l, dl,vel, eps, deps
       dt=0.5

       t=0.
       l=1.
       vel=0.1
       eps=0.

       open(1,file='et_num.txt')
       do while(t.lt.3)
          write(1,'(2f10.5)')t,eps
          dl=vel*dt
          deps = dl/l
c         updates
          l=l+dl
          eps=eps+deps
          t=t+dt
       enddo

       close(1)

       end
```



Another interesting question would be under what velocity we should stretch to have a fixed strain rate?

Find $l(t)$ that gives $\dfrac{d\varepsilon}{dt} = const.$

$$d\varepsilon = \frac{dl}{l} \rightarrow \int_0^{\varepsilon} d\varepsilon = \int_{l_0}^{l} \frac{dl}{l} \rightarrow \varepsilon = \ln\left(1 + \frac{l - l_0}{l_0}\right) \rightarrow \exp(\varepsilon) = 1 + \frac{l - l_0}{l_0}$$

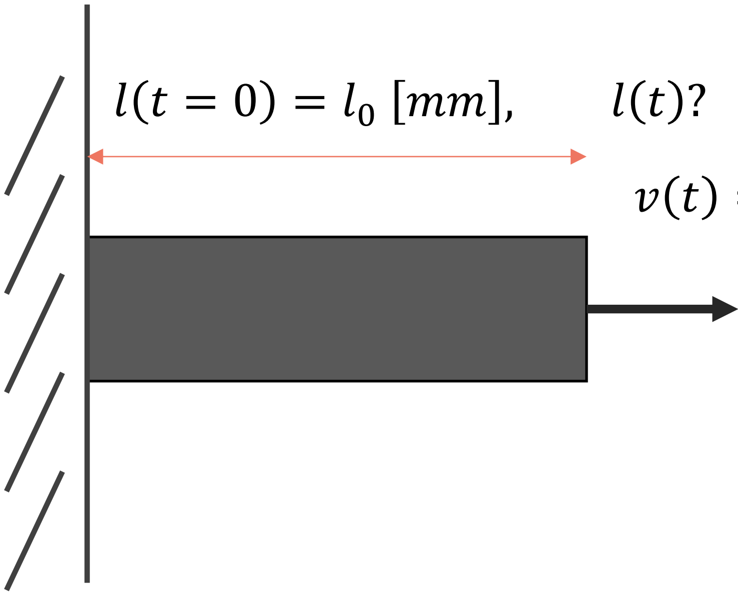
$$\rightarrow l = l_0(e^{\varepsilon} - 1) + l_0 \qquad \rightarrow l = l_0 e^{\varepsilon}$$

$$\rightarrow l(t) = l_0 e^{\varepsilon(t)} \qquad \rightarrow l(t) = l_0 e^{ct}$$

$$FYI, \qquad \frac{dl}{dt} = l_0 c e^{ct}$$

$$d\varepsilon = c\, dt \rightarrow \int_0^{\varepsilon(t)} d\varepsilon = c \int_0^{t} dt \qquad \varepsilon(t) = ct$$

Stretching a one-dimensional rod by a variable speed



$$v(t) = dl/dt = \cos(t\ [\times \mathrm{rad/sec}])\ [\times mm]$$

$$\frac{dv}{dt} = -\sin t$$

Extension – fundamental solution

- The problem can be mathematically expressed as
  $l \equiv l(t)$ with $l(t=0) = l_0[mm]$ and $\frac{dl}{dt} = \cos(t)$
- Express the length as function of t: $l(t)$?

$$l(t) = l(t=0) + \int_0^t dl \quad = l_0 + \int_0^t \frac{dl}{dt} dt$$

$$= l_0 + \int_0^t \cos t \, dt = l_0 + \sin(t)\Big|_0^t = l_0 + \sin(t)$$

True strain as function of time?

$$d\varepsilon = \frac{dl}{l}$$

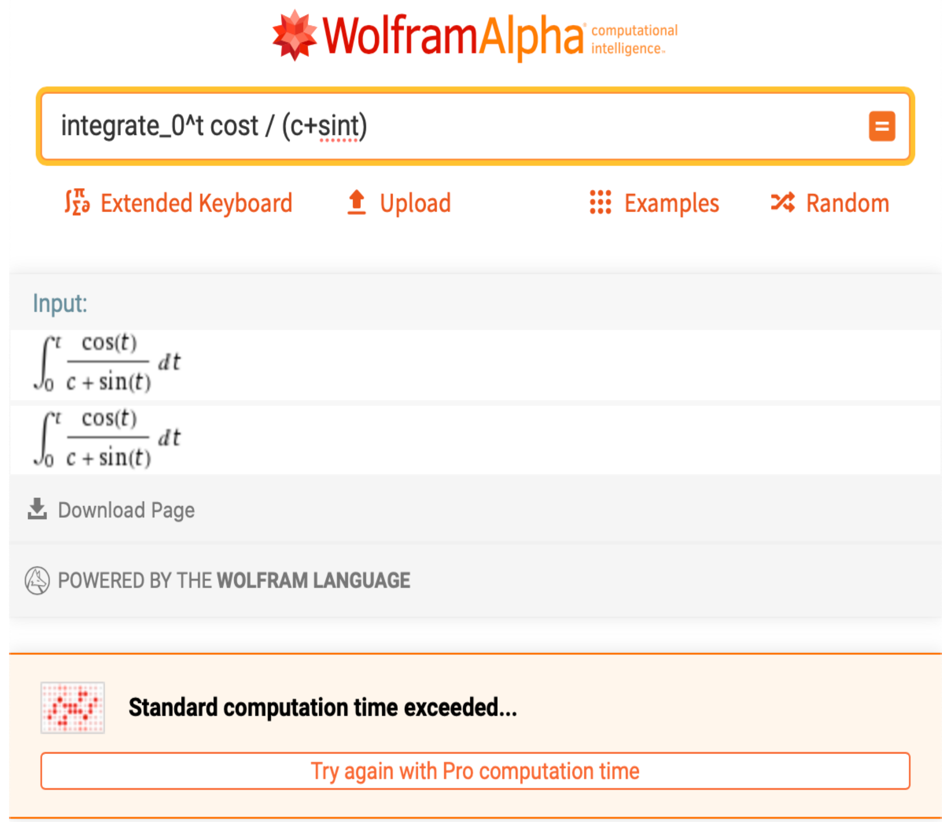
Since $l \equiv l(t)$,

$$d\varepsilon(t) = \frac{dl}{l(t)}$$

As such,

$$\varepsilon(t) = \varepsilon(t=0) + \int_0^t d\varepsilon = 0 + \int_0^t \frac{dl}{l}$$

$$= \int_0^t \frac{1}{l}\frac{dl}{dt}dt = \int_0^t \frac{1}{l_0 + \sin(t)}\cos(t)dt = ??$$



Let's get $\varepsilon(t)$ (numerical approach)

$$d\varepsilon = \frac{dl}{l}$$

$$\frac{dl}{dt} = \cos t$$

$$\varepsilon = \varepsilon(t)?$$

$$\Delta\varepsilon = \frac{\Delta l}{l}$$

$$\frac{\Delta l}{\Delta t} = \cos t$$

$$\Delta l = \cos t \ \Delta t$$

$$\Delta\varepsilon = \frac{\cos t}{l}\Delta t$$

$$t_{(n+1)} = t_{(n)} + \Delta t$$

$$l_{(n+1)} = l_{(n)} + \Delta l$$

$$\varepsilon_{(n+1)} = \varepsilon_{(n)} + \Delta\varepsilon$$

$t_{(0)} = 0$

$l_{(0)}$ is specified

$\varepsilon_{(0)} = 0$

Quite customarily, we try a fixed value Δt

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$$\Delta l = \cos t_0 \, \Delta t$$

$$\Delta \varepsilon = \frac{\cos t_0}{l_0} \Delta t$$

$$t_{(1)} = t_{(0)} + \Delta t$$

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$t_{(0)} = 0$

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$l_{(1)} \leftarrow l_{(0)} + \cos t_0 \, \Delta t$

$t_{(1)} \leftarrow t_{(0)} + \Delta t$

$\varepsilon \leftarrow \varepsilon + \frac{\cos t}{l} \Delta t$

$l \leftarrow l + \cos t \, \Delta t$

$t \leftarrow t + \Delta t$

Let's get $\varepsilon(t)$ (numerical approach)

$t_{(0)} = 0$

$l_{(0)}$ is specified

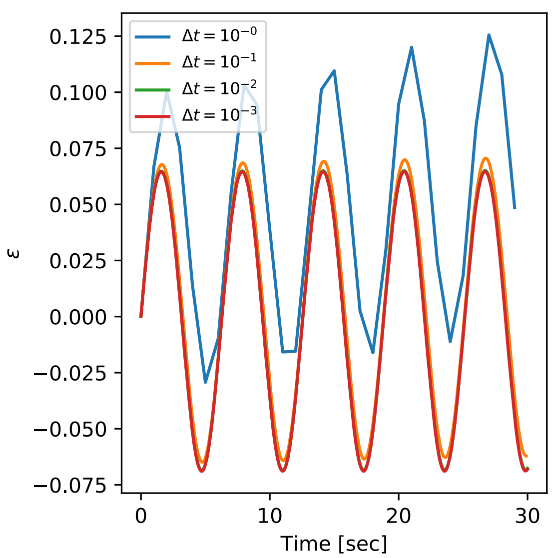
$\varepsilon_{(0)} = 0$

$$\varepsilon \leftarrow \varepsilon + \frac{\cos t}{l} \Delta t$$

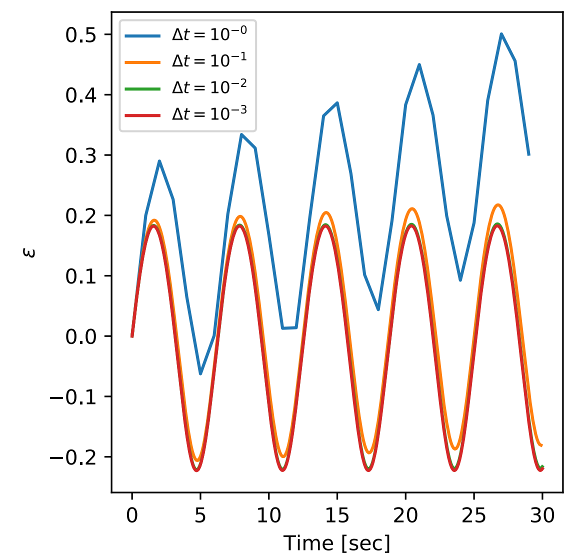
$$l \leftarrow l + \cos t \, \Delta t$$

$$t \leftarrow t + \Delta t$$

For initial length 15



For initial length 5



$\frac{dl}{dt} = \cos t$ (cheat sheet)

$$\varepsilon_{(1)} \leftarrow \varepsilon_{(0)} + \frac{\cos t_0}{l_0} \Delta t$$

$$l_{(1)} \leftarrow l_{(0)} + \cos t_0 \, \Delta t$$

$$t_{(1)} \leftarrow t_{(0)} + \Delta t$$

```
      program var_vel
      implicit none
      character*12 cdt
      integer i
      real eps, t, l, dt

c     initial condition
      l=5.
      t=0.
      eps=0.
      dt=0.01

      open(3,file='var_vel.txt')
      do while(t.lt.30)
         write(3,'(3f10.5)')  t,l,eps
         eps=eps+cos(t)/l * dt
         l = l + cos(t)*dt
         t = t + dt
      enddo
      close(3)

      end program
```

Stretching a one-dimensional rod by a variable speed (#Exercise)