Introduction to computational plasticity using FORTRAN

Youngung Jeong

Changwon National Univ.

Viscoelasticity

- Material that has both viscosity and elasticity

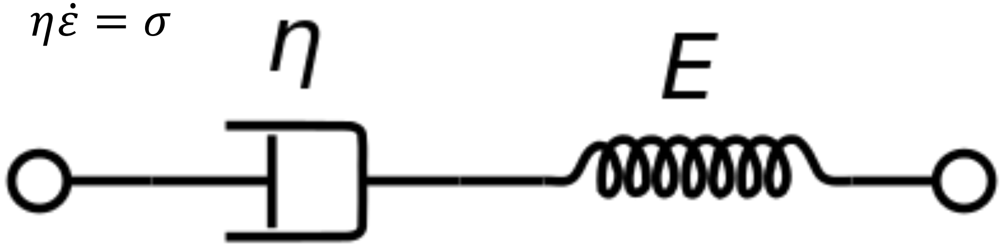
One dimensional elastic rod

- Elastic constitutive law:
  - $\mathbb{E}\varepsilon = \sigma$ (elastic stiffness $\mathbb{E} = 200$ [GPa])
- Viscosity constitutive law:
  - $\eta\dot{\varepsilon} = \sigma$
- Elasticity law: $d\varepsilon^{el} = \frac{1}{\mathbb{E}}\, d\sigma$
- Viscosity law:$\eta \frac{d\varepsilon}{dt} = \sigma$

Maxwell model

$\mathbb{E}\varepsilon = \sigma$

$\eta\dot{\varepsilon} = \sigma$



Total strain is decomposed to viscosity and elasticity

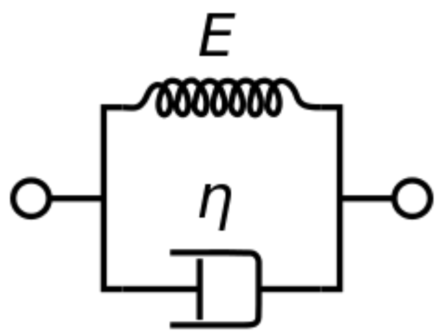
$$\varepsilon^{total} = (elastic\ strain) + (viscosity\ strain)$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \frac{d\varepsilon^{el}}{dt} + \frac{d\varepsilon^{v}}{dt} = \frac{d\sigma}{\mathbb{E}dt} + \frac{\sigma}{\eta}$$

$$\sigma = \eta\dot{\varepsilon} = \frac{\eta\dot{\sigma}}{\mathbb{E}} + \sigma$$

https://en.wikipedia.org/wiki/Viscoelasticity

Kelvin-Voigt model



Total stress is contributed by viscosity and elasticity

$$\sigma = (elastic\ stress) + (viscosity\ stress)$$
$$\sigma = \mathbb{E}\varepsilon + \eta\dot{\varepsilon}$$

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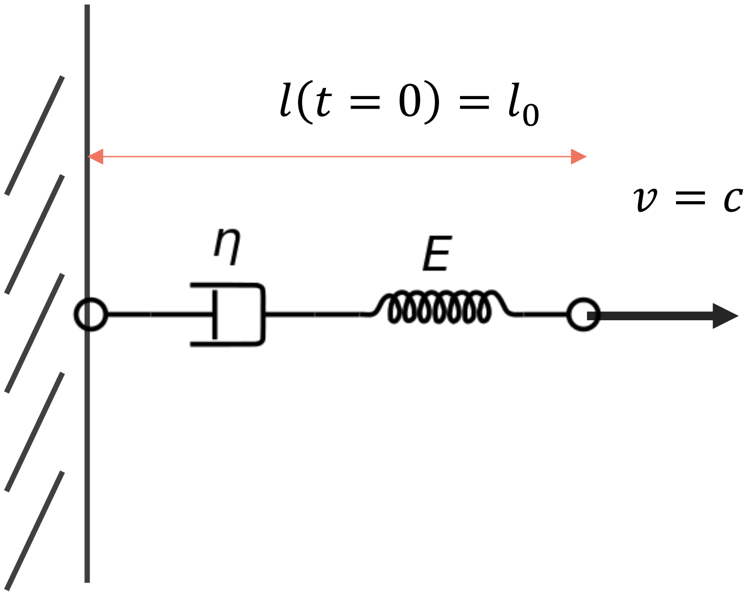
L vs σ? For the case of Maxwell



$$\frac{dv}{dt}=0$$

$$\eta\dot{\varepsilon} = \frac{\eta}{\mathbb{E}}\dot{\sigma} + \sigma$$

$$l = l_0 + ct$$

$$d\varepsilon = \frac{dl}{l}$$

$$\frac{dl}{dt} = c$$

$$l = l_0 + ct$$

$$\frac{d\varepsilon}{dt} = \frac{1}{l}\frac{dl}{dt}$$

$$\rightarrow c\frac{\eta}{l} = \frac{\eta}{\mathbb{E}}\frac{d\sigma}{dt} + \sigma$$

$$\rightarrow c\frac{\eta}{l_0 + ct} = \frac{\eta}{\mathbb{E}}\frac{d\sigma}{dt} + \sigma$$

Can we get $\sigma = \sigma(t)$?

Yes, numerically… (exercise)