02. Physical quantities represented in Cartesian coordinate system

강의명: 금속유동해석특론 (AMB2039)

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Objectives

- Understand material property

- When defining a material property, one has to look for a pair of stimulus (자극) and response (반응).

- When you have enough of ‘knowledge’ (experimental data), you can correlate the stimulus to the response in a ‘linear’ basis.
  - That is, Response (as a quantifiable value) = Property times Stimulus (as a quantifiable value)
  - A straightforward example is:
    - Stress = Modulus times Strain
    - Strain = compliance times Stress

Outline

- Mathematical representation of physical quantity
  - Scalar
  - Vector
  - Tensors
- Cartesian Coordinate system
- Coordinate transformation
  - Rules to transform scalar
  - Vectors
  - Tensors
- Material properties represented as tensors
- Physical rules represented by tensors

Physical quantities

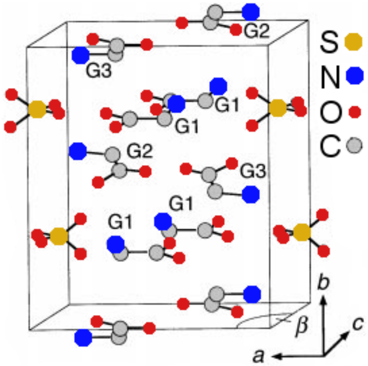
- Q) What is a physical quantity?
  - * A physical quantity is a physical property of a phenomenon, body, or substance, that can be quantified by measurement.
- Mechanical "physical quantities" that we are interested in:
  - Displacement, force, and velocity and so forth.
- Universe looks the same to all observers, regardless of how they move (relativity)
  - That means, a physical quantity should not be affected by 'coordinate system'.
  - That also means, that the physical rules (laws) should not be affected by 'coordinate system'.
  - We will learn how this can be achieved by representing physical quantities using **tensor**.

Direction dependence in physical quantities

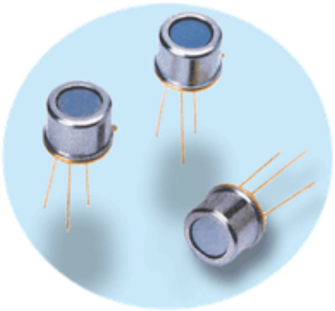
- Some physical quantities are direction-dependent.
- Some material properties are direction-dependent.
- How to represent the properties (and other physical quantities such as response/stimulus) that are direction-dependent?
  - Scalar
  - Vector
  - Tensor?
- 0^{th} rank tensor is scalar
- 1^{st} rank tensor is vector
- There can be 2^{nd} rank tensor, 3^{rd} and so forth.

Application: motion sensors

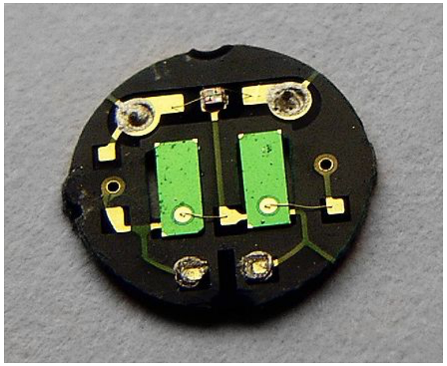
- You can easily find lights that are controlled by motion sensors at your home, rest rooms and so forth.
- Many motion sensors have pyroelectric (pyro: temperature) materials such as lead titanate ($PbTiO_3$) and triglycine sulphate.



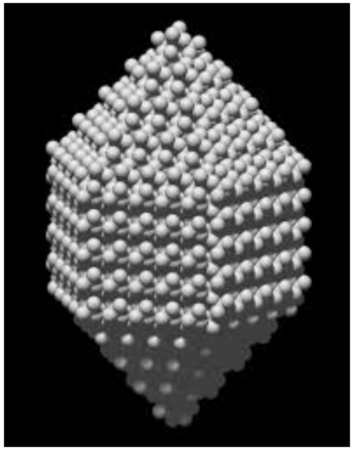
- Variable (or stimulus) is the heat input to the material
- Response of the material under such a stimulus is electric polarization
- They can be linearly correlated through a certain set of numbers:



- $\vec{\mathbf{p}} = \frac{\partial \vec{\mathbf{P}}}{\partial \mathrm{T}}$ where $\vec{\mathbf{p}}$ (lowercase) is the coefficient (slope)
- and $\vec{\mathbf{P}}$ (uppercase) is the polarization **vector.**
- A vector has 'three' components:
- $p_1 = \frac{\partial P_1}{\partial T}, p_2 = \frac{\partial P_2}{\partial T}, p_3 = \frac{\partial P_3}{\partial T}.$
- The above is shortened to a form: $p_i = \frac{\partial P_i}{\partial T}$ (where the subscript i=1,2,3)



Application: quartz oscillators



Quartz crystal exhibits an interesting material property:
When the quartz crystal is exposed to electricity (electric field), the crystal *distorts* (strain);
The stimulus and the response can be linearly correlated through:

$$\varepsilon_{ij} = d_{ijk}E_k$$

This notation with subscript will be discussed later (Einstein summation convention). Yes, it is the physicist.

What to be discussed

- Response = Property x Stimulus
- Linear properties
- Axis transformation (coordinate transformation; changing the coordinate system; it does not change the physical quantity)
- Scalars, vectors, and tensors
- Tensor transformation rule
- Examples

Math used to relate microstructure-property

- Material property is defined on the basis of 'stimulus'-'response' pairing.
- A stimulus is something that one does to a material: e.g., load (force)
- A response is something that is the result of applying a stimulus:
  - If you apply 'force' the material will change its shape (strain)
- The material property is the 'connection' between the stimulus and the response.
- The material property is 'quantifiable' between different types of physical quantities (scalar, vector, tensors and so forth).
- In fact, scalar and vector is a specific type of 'tensors' and tensor is the generalized way to express the stimulus, response and even the material properties.

Stimulus -> Property -> Response

- Mathematical framework to describe the connection among stimulus, property and response?
- The property is equivalent to a function (P) and the stimulus(F) and response (R) are variables. The stimulus is also called a field because in many cases, the stimulus is actually an applied electrical field or magnetic field, or pressure field, or force of some kind.
- The response (R) is a function of the field so we insert the symbol P to designate the material property:
- $R=R(F)$
- $R=P(F)$

Scalar, linear properties

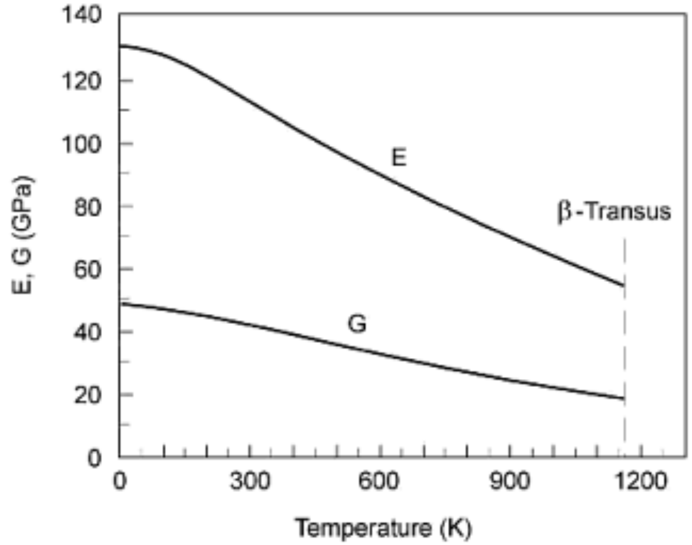


Fig. 2.4. Modulus of elasticity E and shear modulus G as a function of temperature of α titanium polycrystals [2.2]

- In many instances, both stimulus and response are *scalar* quantities, meaning that you only need one number to prescribe them. Specific Heat is an example of a scalar property.

- To further simplify, some properties are linear, which means that the response is linearly proportional to the stimulus: $R = P \times F$. However, the property is generally dependent on other variables.
- Example: elastic stiffness in tension/compression changes with, or is a *function of temperature*, which we indicate by adding "*(T)*" after the symbol for the property, "*P*":

$$\sigma = E(T) \times \varepsilon$$
$$\equiv R = P(T) \times F.$$

Example above shown of Young's modulus and Shear modulus versus temperature for Ti-6Al-4V, courtesy of Brian Gockel

Scalars, vectors, tensors

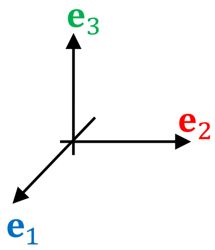
- *Scalar*:= quantity that requires only one number, e.g. density, mass, specific heat. Equivalent to a zero-rank tensor.
- *Vector*:= quantity that has direction as well as magnitude, e.g. velocity, current, magnetization; requires 3 numbers or *coefficients* (in 3D). Equivalent to a first-rank tensor.
- *Tensor*:= quantity that requires higher order descriptions but is the same physical quantity, no matter what coordinate system is used to describe it, e.g. stress, strain, elastic modulus; A general concept to mathematically express the physical quantities and the associated material properties.

Anisotropy

- *Anisotropy* as a word simply means that something varies with direction.
- Anisotropy is from the Greek: *aniso* = different, varying; *tropos* = direction.
- Almost all crystalline materials are anisotropic; many materials are engineered to take advantage of their anisotropy (beer cans, turbine blades)
- Older texts use trigonometric functions to describe anisotropy but tensors offer a general description with which it is much easier to perform calculations.
- For materials, we know that some properties are anisotropic. This means that several numbers, or *coefficients*, are needed to describe the property - one number is not sufficient fully quantify the anisotropic property.
- Elasticity is an important example of a property that, when examined in single crystals, is often highly anisotropic. In fact, the lower the crystal symmetry, the greater the anisotropy is likely to be.
- *Nomenclature:* in general, we need to use *tensors* to describe fields and properties. The simplest case of a tensor is a *scalar* which is all we need for isotropic properties. The next "level" of tensor is a *vector*, e.g. electric current.
- Q) Which one do you think is more anisotropic: cubic or hexagonal-closed-packed. Why?

Coordinate system and basis vectors

- 앞으로 좌표계를 설명할때 좌표계의 근간이 되는 방향들을 normal vector (즉 크기가 1인 벡터)로 표현.
- Cartesian coordinate system은 orthonormal coordinate system
- 서로 수직한 세 normal vector로 표현이 가능하다.



그 세 normal vector들을 basis vector로 칭하겠다.
그리고 각각 $\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3$ 로 나타내겠다.

가령, vector ***v***는 위의 basis vector에 평행한 성분들로 구성할 수 있다.

$$\boldsymbol{v} = v_1\mathbf{e_1} + v_2\mathbf{e_2} + v_3\mathbf{e_3}$$

Letters in boldface is vector;

Physical law and material properties

- Stress and strain are ‘linearly’ correlated for most metals within their elastic regime.
  - Q: How is stress related to strain?
  - Q: Can you explain Hooke’s law?

Tensor and Matrix (행렬)

- The first confusion I encountered when I learned tensor was mostly due to the confusion of the tensor with matrix.
  - It is because I learned 2nd rank tensor, and the 2nd tensor is generally written in a 3x3 matrix.
- Remember: matrix is a method to represent certain values in a table and is so valid only for 2nd rank tensors (or 2 dimensional tensors consisting of single row and single column)

- **Tensors are what transform like tensors**
  - By learning the way tensor 'transforms', you'll learn the most important aspect of tensor.
  - We will learn about this transformation rules applied for tensors later.

- Einstein used Tensors:
  - "In the 20th century, the subject came to be known as *tensor analysis*, and achieved broader acceptance with the introduction of Einstein's theory of general relativity around 1915. General relativity is formulated completely in the language of tensors. Einstein had learned about them, with great difficulty, from the geometer Marcel Grossmann. "

Recap

- Material property ’connects’ the stimulus and its paired response.
- For 1D linear:
  - R=P x S
- Scalar and vectors are just two special cases of tensors – they are 0 rank and 1st rank tensors, respectively.
- For the case of ‘anisotropic’ and linear (this actually will be learned later):
  - $R = P\ S$
  - $R_{ij} = P_{ij}S$
  - $R_{ij} = P_{ijk}S_k$
  - $R_{ij} = P_{ijkl}S_{kl}$ .. And more. (We’ll learn about this conventions later)

References and acknowledgements

- References.

- Acknowledgements

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