Orientation Distribution and Pole Figures

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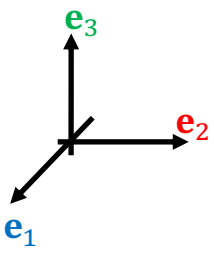
Coordinate system and basis vectors

- 여러분들은 x,y,z 축 등의 기호를 사용하여 그리진 좌표계에 익숙할 것이다.
- 앞으로 좌표계를 설명할때 좌표계의 근간이 되는 방향축들을 normal vector (즉 크기가 1인 벡터)로 표현하고자 한다.
- Cartesian coordinate system은 orthonormal coordinate system
- 서로 수직한 세 normal vector로 표현이 가능하다.



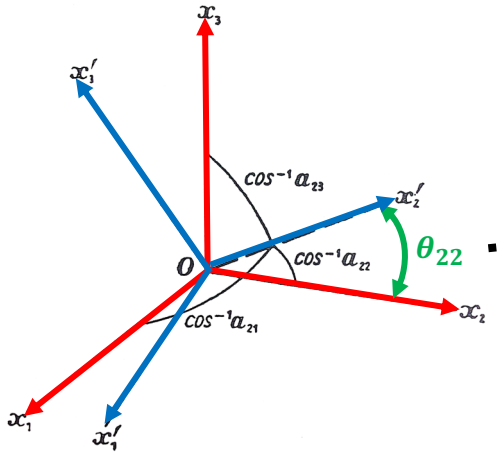
그 세 normal vector들을 basis vector로 칭하겠다. 그리고 각각 e_1, e_2, e_3 로 나타내겠다.

앞으로 다루게 될 응력과 변형률 텐서의 성분(component)을 표현할때 쓰이는 subscript 인덱스 (1,2,3)는 각각 e_1, e_2, e_3 벡터들을 뜻한다.

방위의 수학적 표현 방법

- Quaternion (convenient mathematical notation for representing orientations)
- Rodriguez (a vector and an angle; any arbitrary orientation; misorientation)
- Euler angles (Bunge notation (ϕ_1, Φ, ϕ_2) is widely used – ZXZ convention)
- Transformation matrix

두 축 (axes) 간의 방위 관계



위 그림에서는 x_1, x_2, x_3 로 이루어진 좌표계1과 $x`_1, x`_2, x`_3$로 이루어진 또 다른 좌표계2가 나타나있다. 좌표계 2의 $x`_2$ basis vector와 좌표계 1의 각 basis vector들과의 관계를 a_{21}, a_{22}, a_{23} 의 direction cosine으로 표현했다.

$$a_{22} = cos(\theta_{22})$$

Old co. sys.

$$[\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]$$

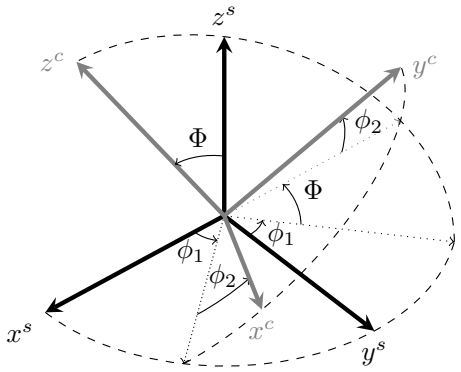
New co. sys.

$$\begin{bmatrix} \acute{\mathbf{e}}_1 \\ \acute{\mathbf{e}}_2 \\ \acute{\mathbf{e}}_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Transformation matrix (not tensor)

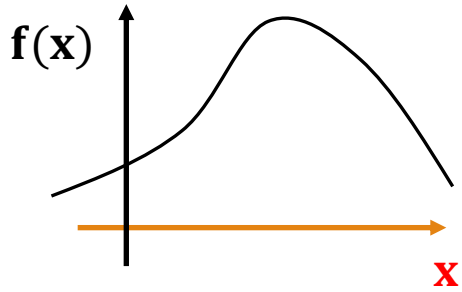
서로 다른 두 좌표계(axes, coordinate)의 방위 관계가 하나의 transformation matrix로 표현됨.

A LaTeX script to generate below illustration:
https://youngung.github.io/euler/

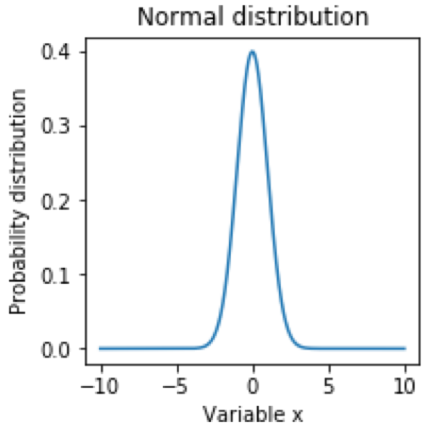


분포 (Distribution)

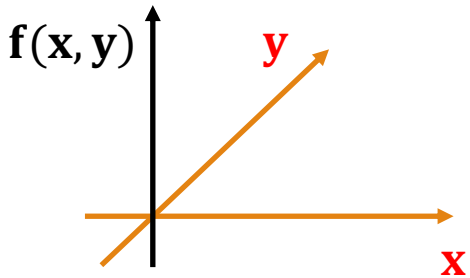
❑1차원 분포 (변수가 하나, univariate)



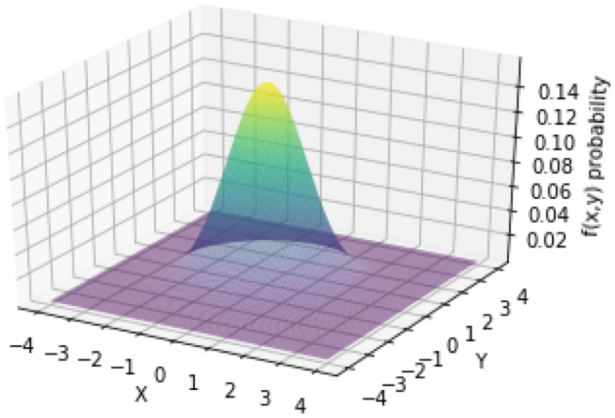
1차원 공간의
변수 (x)



❑2차원 분포 (변수가 두개, bivariate)

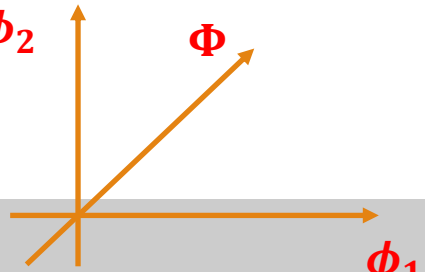


2차원 공간의
변수 (x,y)



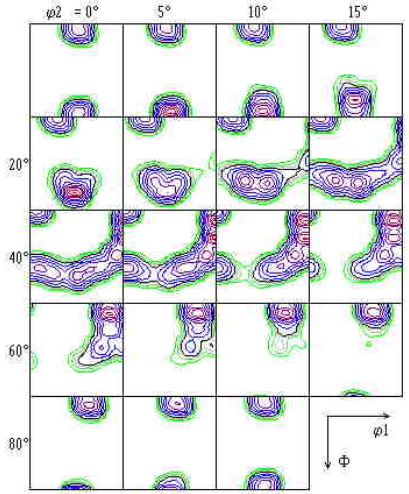
❑3차원 분포

➢Euler angles 로 표현된 방위(orientation)은 3차원 분포



3차원 공간의
변수 (ϕ_1, Φ, ϕ_2)

$\mathbf{f}(\phi_1, \Phi, \phi_2)$?



Discretization (불연속화)

- 방위의 분포를 불연속화하여 방위 분포의 sampling 필요.
- 임의 추출법을 활용 (random sampling)
- Randomly and repeatedly sample discrete orientations and give properly weights determined by orientation distribution.

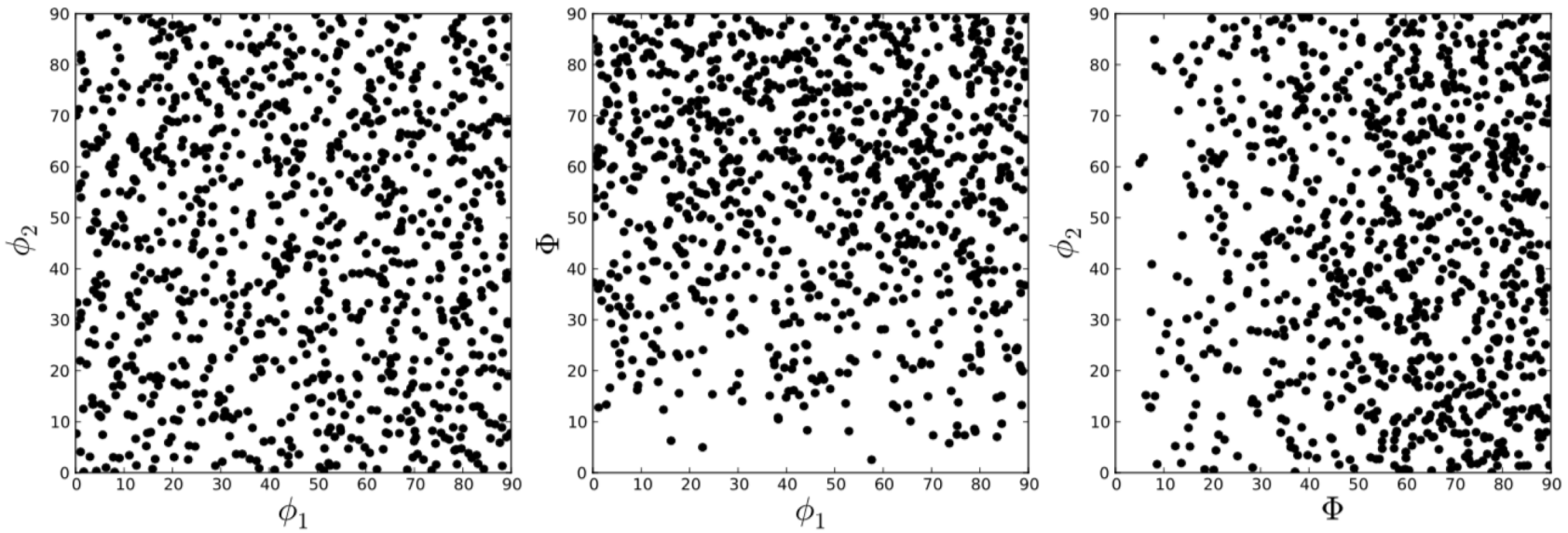
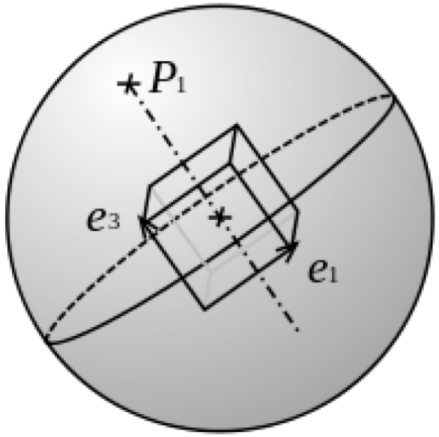
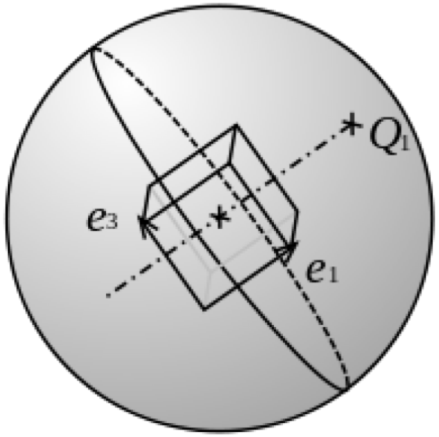


Figure 1. Projections of the 1000 grains sampling points on the planes perpendicular to Φ, ϕ_2 and ϕ_1 axes, respectively.

Pole figure



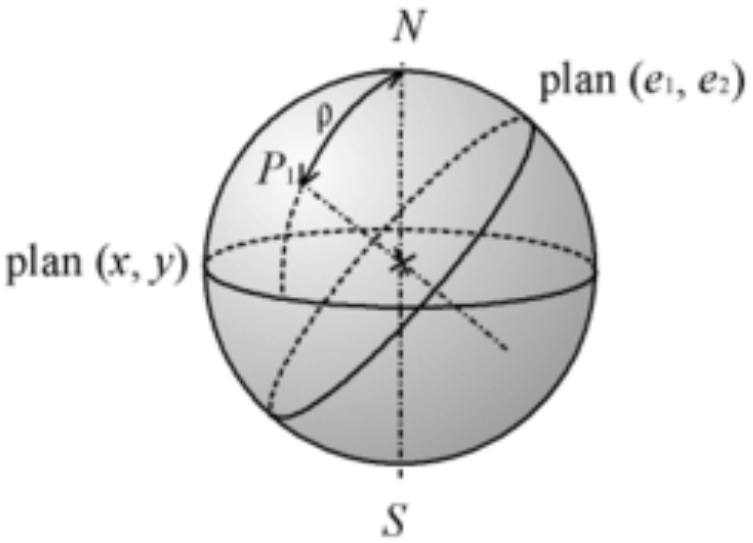
P_1 : pôle du plan (e_1,e_2) - plan (001)



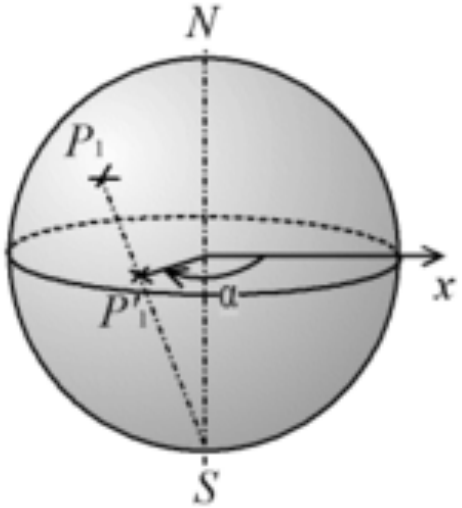
Q_1 : pôle du plan (e_2,e_3) - plan (100)

https://en.wikipedia.org/wiki/Pole_figure

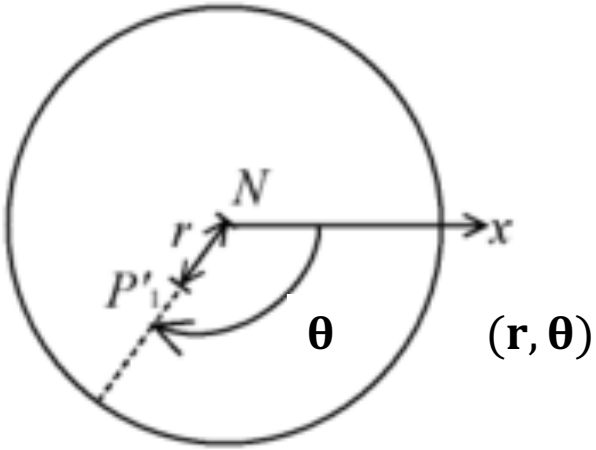
Pole figure



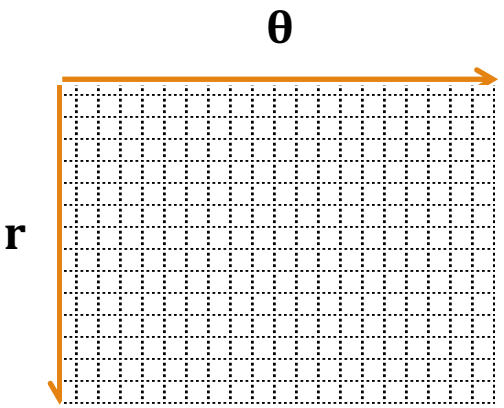
Vector to spherical coordinates



Stereographic projection



$(\mathbf{r}, \boldsymbol{\theta})$



https://en.wikipedia.org/wiki/Pole_figure